Exercise 88

Among all of the pairs of numbers whose sum is 6, find the pair with the largest product. What is the product?

Solution

Let x and y be two real numbers whose sum is 6.

$$x + y = 6$$

Solve for y.

$$y = 6 - x$$

Now take the product of x and y, substitute the formula for y, and complete the square to write the quadratic function in vertex form.

$$P = xy$$
$$= x(6 - x)$$
$$= 6x - x^2$$
$$= -(x^2 - 6x)$$
$$= -[(x^2 - 6x + 9) - 9]$$
$$= -[(x - 3)^2 - 9]$$
$$= -(x - 3)^2 + 9$$

Therefore, the maximum product is $P = 9$, which occurs when $x = 3$ and $y = 6 - 3 = 3$.